

CHAPTER VII

ORTHOPEDAGOGIC-ORTHODIDACTIC EVALUATION OF AND ASSISTANCE FOR CHILDREN WITH ARITHMETIC PROBLEMS

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1. Introduction

In light of the views expressed in the previous chapters, the following is once again stressed: orthopedagogic-orthodidactic evaluation and assistance do not merely involve the diagnosis and "remediation" or correction of arithmetic problems. To view a problem with learning arithmetic merely as a subject matter difficulty or as an isolated defect and then to diagnose (by an analysis of errors) and to "remediate" this difficulty (by an "extra dosage" of arithmetic as a treatment of the symptoms), is to misunderstand and overlook the **child** with arithmetic problems, as pedagogically situated, as an existential totality, as a person (**experiencing*** I), as a being in relationship (child-in-the-world).

Arithmetic difficulties, at most, are only symptoms of deeper pedagogic, didactic or psychopedagogic problems (e.g., pedagogic or didactic neglect); consequently, the problem of the child with arithmetic difficulties implicates his total pedagogic and didactic situations as well as the ways he **experiences** them (as his experiential world of learning), and, therefore, evaluating and correcting these difficulties clearly are orthopedagogic-orthodidactic matters. Thus, in the present chapter the point of departure is the pedagogic situation of the child with arithmetic problems and the primary method is a phenomenological penetration of his learning-arithmetic world (as experiential world). Questions guiding this reflection are how the child **experiences** his failures in arithmetic; i.e., what does he think, feel and strive for, and what value or importance does he attribute to his failures?

A child doesn't **experience** his failures in school only with his intellect but as a total person and, therefore, his failures involve his

* See previous chapters regarding the translation of "beleweniswereld" and "belewing".

total existence. Thus, assisting such a child should not be directed only to his schoolwork (his intellectual aspect) but also to the pathic-affective and normative **experiences** of his problems with learning arithmetic.

A child's problems with arithmetic are life problems that influence his entire child-world relationship and, consequently, his total becoming as a person. Therefore, such a child is viewed in terms of how he **experiences** his problems (by orthopedagogic-orthodidactic evaluation), and he is supported in his **experiences** and experiential world (by orthopedagogic-orthodidactic assistance). Thus, the orthodidactic (arithmetic) question also is an orthopedagogic one: How must the educator support this child with arithmetic problems in his **experiences** and experiential world so that he can attain full-fledged adulthood (also via his arithmetic achievements)?

Arithmetic (learning) derailments also mean for a child a derailment in his being on-the-way to adulthood and this is why arithmetic difficulties are not a purely orthodidactic or "remedial" teaching matter. On the one hand, according to the norms, standards and requirements of the school, the child **has** an arithmetic problem; on the other hand, there is the child's subjective **experiencing** of his arithmetic problems, of his distress and of his failure. Thus, the arithmetic problem also is something **experienced** and, hence, it is an orthopedagogic matter that involves the re-education or corrective educating of the child who, with respect to his learning-arithmetic, has become derailed in his becoming adult. Clearly, a child with arithmetic problems is an orthopedagogic-orthodidactic task.

2. The nature of the arithmetic system

According to Van Gelder⁽¹⁾ arithmetic is an activity in relation to the quantitative aspect of reality that is aided by an arithmetic system existing in the culture. Teaching arithmetic, then, is creating situations* within which a child acquires new experiences (in the field of quantitative relations) so that he can be guided to an independent command of the arithmetic system.⁽²⁾ Attaining insight

* For examples of such situations, the reader is referred to Van Gelder, L., **Grondslagen van de Rekendidaktiek**, p. 26.

into numbers and number relations by the child is one of the most important tasks of arithmetic didactics.⁽³⁾

Arithmetic originates in the communication among persons and in the fact that in our culture there are buying, calculating and bartering. Thus, to be able to calculate is a cultural skill. In a culture where there is no calculating, no arithmetic problems will ever be discovered.

The concern here is with the child's relationship to (and **experiencing** of) the quantitative world (the quantitative aspect of reality). The quantitative world that has to be acquired by the child will be viewed in the present chapter as an experiential world.

By **quantitative world** is understood the total conglomerate of numbers, number concepts, number relations, quantity, quantity relations, number notations, arithmetic concepts, fractions, solution methods, operations, operation signs and schemes, ordination, automatisms (e.g., times tables and combinations), language sums or arithmetic problems (the most difficult task in arithmetic), space and time relations, arithmetic structures, etc. This world has to be mastered and acquired by the child so that he can deal with, order and command it.

To be able to calculate means to know **which** number operations to use in a particular situation to arrive at the desired result⁽⁴⁾; also, one has to be able to carry out the operations well (The former task is the more difficult one). Working with numbers often is not the problem but rather interpreting the text (problem) which gives rise to the number operations.⁽⁵⁾

To compute, a child has to have at his disposal at least:

- a) an understanding of quantity and of the relationships more/less, larger/smaller, part/whole;
- b) the number as an indicator of quantity;
- c) the notation system.⁽⁶⁾

The arithmetic system has the following characteristics:

- a) the spoken series of counting (as an open system);

- b) the notations (number notations, operation notations and schemes);
- c) arithmetic problems.

An unsatisfactory command of language already points to the origin of difficulties with the spoken series of counting; a second set of difficulties is entered when the child has to learn to command the notation system and do arithmetic problems in the elementary school. Rightly, this is a serious didactic problem because for a child this is the most difficult arithmetic task.⁽⁷⁾

Preconditions for a child's acquisition of this intricate system of notations and operational schemes are his dealings with things, his possession of language, and his transition from a pathic-affective to a gnostic-cognitive attunement.⁽⁸⁾ Becoming proficient in arithmetic is a slow and gradual event. In this respect, Van Gelder⁽⁹⁾ distinguishes the following three phases:

- a) **a non-arithmetic phase:** the child has some numerals at his disposal but can't use them appropriately;
- b) **a pre-arithmetic phase:** the child slowly expands his knowledge of numerals; he uses them for counting in life situations; he also is able do simple addition;
- c) **an arithmetic phase:** the child is able to command the number relations in technical ways.

3. The nature of arithmetic as a school subject

a) Arithmetic is the only elementary school subject that keeps the child engaged with the quantitative. In arithmetic, the concern is with quantities and especially with the numbers and notations used to express particular quantities. Numbers are abstract concepts that are represented by number-symbols. The concept of quantity has always existed, but the number system and our computational systems are (more recent) creations of human intellect. Drummond (Psychology and teaching of number) touches the core of the problem of arithmetic when he says, "Arithmetic differs from most subjects in that the hardest step of all has to be taken at the very beginning. At the very beginning the child has to leave the world of concrete fascinating realities and concentrate on an abstraction, on a creation of the human intellect." This abstract nature of

arithmetic is one of the primary reasons why such a large number of pupils experience learning problems in this subject.

b) Arithmetic is a logical subject. Because it is, the steps follow each other logically. This implies that knowledge of a certain step is necessary as a prerequisite for subsequent steps. A deficient knowledge of the basic concepts has the consequence that the child is in a position to understand the ensuing concepts. In a subject such as history such a logical order of concepts doesn't exist and one fact is not necessarily built upon preceding ones. A deficient knowledge of a certain part of history need not affect a child's achievement in another part. In arithmetic this is not the case because often various basic principles and operations are required for solving a problem. It is not unusual for a pupil to need to know the main operations, fractions, decimals, percentages, relations, proportions, etc. to solve a particular problem. In the lower grades one operation is built upon another. For example, multiplication is presented as the successive addition of the same number, subtraction is dealt with as the inverse of addition and division is considered as the successive subtraction of the same number. The child experiences that by subtracting a multiple of the divisor he can shorten his work. Again, this involves new concepts that lead to acquiring a technique. Deficient knowledge of any of the basic principles will hamper him in forming basic schemes which lead to forming structure and which is necessary for using the knowledge in solving problems.

For the reasons mentioned, it is of utmost importance that there is continuity in the course of learning arithmetic; that is, in the entire course of learning arithmetic the learning material has to be sequenced logically, and, further, there has to be a well-planned articulation of the curricula for the different grade levels. If the work prescribed for a particular grade level is not completed during the year, it can be expected that the pupil will run aground with problems during the ensuing year because he will not have the necessary knowledge to solve them. Also, he will not be able to progress with particular topics before the work missed in the previous grade is taught. This problem also can occur in cases of lengthy absences from school. Since each school also plans its own work scheme, this problem also can arise by changing schools. In changing schools it sometimes happens that a pupil repeats a certain part of the work while another part is not done. This is

usually one of the reasons pupils who are absent a great deal or who change schools a lot experience learning problems in arithmetic.

c) A preschool child is already informally acquainted with the arithmetic system. This usually takes the form of incidental learning. A preschool toddler learns in natural ways through discussing, playing, imitating and doing. In the preschool period, play has an important role in the child's becoming. During this period, arranging quantities and the use of language relevant to this arise. Also, he becomes acquainted with and has experiences with space, size, form, weight, length, comparisons, etc. These experiences are necessary for arithmetic to be meaningful later. At this stage, the necessary language also is acquired because children continually talk about what they do. This acquaintance with the arithmetic system during the preschool period occurs on a concrete-visual level. These contents furnish the perceptual layer of thought.

d) The attunement or disposition awakened by the arithmetic system during the first year of school and even during the preschool is very important because this can largely determine the pupil's achievement in arithmetic.

In school, incidental learning now makes way for more intentional learning and, consequently, a child's directedness becomes all-important. Now he consciously learns. His concrete dealing need to lead to forming a model and later an etiquette scheme which results the act-structures that develop into automatisms. Then, these automatisms will be based on meaningful insights and not mere memory.

e) It is an acknowledged fact that children first have to be ready for arithmetic before they can acquire anything of value from formal instruction in arithmetic. This readiness should not be equated with a particular chronological age because all children differ from each other. Sonnekus⁽¹⁰⁾ differentiates between school maturity and school readiness. School readiness is a "growth process" within the child and is primarily biological. School readiness refers more to a total fitness for school and is directly influenced by the educating he receives from his parents. Rienstra⁽¹¹⁾ also emphasizes the detrimental effect that formal school instruction has on the "immature" school beginner. Vedder⁽¹²⁾ lists the following characteristics of a child who is ready for school:

- i) he must be able to accept and carry out instructions. Thus, he must be able to control and command himself;
- ii) he must be able to limit his attention to the instruction;
- iii) he must be directed to learning;
- iv) he must be able to control his impulses;
- v) he must be able to communicate with his teachers and classmates;
- vi) he must be able to follow rules.

These characteristics also hold for a child who is ready for arithmetic instruction.

Arithmetic readiness should be viewed as a progressive event in the sense that a pupil has to continually become ready for the following step or grade level. Thus, it is a teacher's obligation to continually make his pupil's ready for the next step. Van Gelder⁽¹³⁾ also discusses three stages of arithmetic readiness:

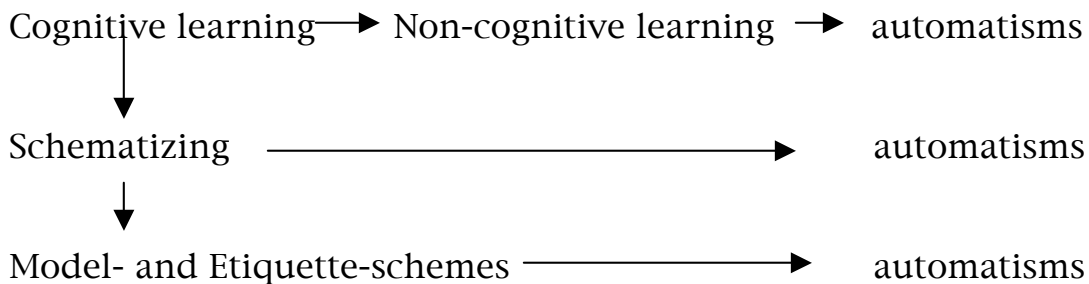
- i) from 3-5 years, during which the language for referring to quantity is acquired;
- ii) from 5-8 years, which is characterized by the effective use of numbers and the application of operations to concrete quantities;
- iii) after 8 years, which is characterized by an intelligent insight into number relations. At this stage, the role of language is very important because a certain vocabulary is required before a child can understand any oral or written problems. Thus, preparatory and beginning arithmetic, in the first instance, has to be language training.

During preparatory arithmetic, manipulating concrete objects is important for forming concepts that furnish the concrete-visual level of thinking. Schonell⁽¹⁴⁾ also emphasizes that a child should not become acquainted with symbols too quickly. When he has lived through sufficient meaningful concrete experiences, it will be easier to make a transition to forming abstractions.

f) Modes of transition from the concrete to the abstract have to be formed. The three levels of thought, namely, the concrete-visual, the schematic and the abstract should always be kept in mind. According to the German psychology of thought, these three levels constitute a structural whole and should not be separated.

Kohnstamm also stresses that thinking does not remain static on one level. If a solution cannot be found on one level it is sought on another. He ascribes many errors in thinking to a weak course or exchange among the different levels. This point of departure makes it clear that an arithmetic teacher has to give more importance to the ways pupils think while finding a solution rather than only to the mere solution itself.

Buswell (in *Methods of studying pupil's thinking in arithmetic*) also emphasizes the importance of teachers knowing how children think. Odendall⁽¹⁵⁾ concludes that during the forms of transition, the pupils have to be guided from the concrete or visual to the schematic level of thinking. This schematic level is a bridge between the visual and abstract levels. According to Van Parreren, the essence of this intermediate level is system forming that develops out of schemes (model schemes that are more concrete and etiquette schemes that are more abstract). Automatism can be formed in one or more of the following ways:



By system forming (building up schemes), a child constructs his own arithmetic world and organizes it into something meaningful for himself. Thus, system forming is the transition between cognitive act-structures and non-cognitive act-structures or automatism.

For example, a child has to master the "ten for one borrowing" system as well as the systems of addition, subtraction, multiplication and division.

Where at first a child might have thought as follows:

$$\begin{aligned}
 7 + 8 &= (7 + 3) + 5 \\
 &= 10 + 5 \\
 &= 15
 \end{aligned}$$

Later, he is able to provide an answer immediately. It is important that the child understand and from the beginning also be able to explain how he arrived at an answer. This means the non-cognitive act-structure has to be based on insight and not mere memorization.⁽¹⁶⁾

4. Arithmetic problems of children

With respect to arithmetic problems, Van Gelder⁽¹⁷⁾ makes the following differentiations*:

- a) **Acalculia:** the entire or partial loss of already acquired arithmetic proficiencies. It means to no longer be able to compute. It can appear in an adult, e.g., after brain damage. Acalculia is not an isolated occurrence but always is a subpart of a complicated occurrence of the nature of aphasia, agnosia and apraxia.
- b) **Dyscalculia:** in the first place, children's arithmetic disturbances should be viewed as **disturbances in becoming**. Arithmetic difficulties are **deviations and difficulties in learning arithmetic**, in acquiring arithmetic proficiencies. Thus, here a child has not yet acquired arithmetic proficiencies and he is impeded in doing so by shortcomings in his milieu, affective disturbances or psychic defects. Therefore, he uses the systems acquired only partly or incorrectly. Dyscalculia thus means not-yet-being-able-to-compute.

Two forms of dyscalculia are:

- i) dyscalculia as a deviation in learning to acquire the language forms that make a command of relations in the quantitative sphere possible. This form appears in the preparatory phase of arithmetic. Examples are deficiencies in dealing with concepts such as more, less, just as much, equal, etc.; deficiencies in ordering quantities; confusions in counting, of symbols, etc.;
- ii) dyscalculia as a deviation in the learning or command of the methods of operation with symbols that represent quantitative relationships. This form of dyscalculia manifests itself in systematic arithmetic (computing). Examples are counting or symbol

* Other terms for this phenomenon are arithmetic weakness, arithmasthenia, specific disability in arithmetic, backwardness in arithmetic, arithmetic derailment.

confusions, deficiencies in the command of elementary operations (addition, subtraction, multiplication, division), of the number system and of operational schemes.

For further descriptions of pathological arithmetic disturbances, e.g., such as number deafness, number-reading blindness, number aphasia and number agraphia, the reader is referred to the work of Van Gelder⁽¹⁸⁾, while Grewel⁽¹⁹⁾ differentiates between asymbolic and asemantic arithmetic disturbances as well as between primary and secondary acalculia.

In addition, arithmetic difficulties lie in three subparts⁽²⁰⁾:

a) The solution of problems. Required for this are:

- i) reading the problem;
- ii) arranging the data for necessary computational operations;
- iii) carrying out the operations;
- iv) making connections between the part-results from the operations and the data from the problem.

b) Carrying out the operations (systematic computations). Difficulties that can arise here are:

- i) the operation schemes are carried out incompletely or not at all;
- ii) different operation schemes are confused with each other;
- iii) particular aids are not remembered (e.g., difficult combinations and times tables);
- iv) particular numbers and signs "intrude" on the operation schemes (subtraction, division and zero signs);
- v) self-discovered methods of solution are used instead of the ones taught.

c) Handling the numbers and their simplest relations. We differentiate three additional computational phases that give rise to characteristic forms of dyscalculia during each phase⁽²¹⁾:

- i) In the first phase, computations are closely connected with language such that computational difficulties are on an equal footing with language difficulties (difficulties of a perceptual, temporal and spatial nature);

ii) in the second phase, computations are in the use of solution methods. Defects in the child's gnostic-cognitive structure and the correct use of cognitive means (solution methods) are the difficulty here;

iii) in the third phase, computation is an aid for solving a problem. Here difficulties are the defective command of the methods of solution or an inadequate understanding of the problem read.

5. The relation between language and problems with arithmetic

A child's language plays a very important role in his involvement with the world because it is a means for giving sense and meaning to it. When a child discovers language as spoken-language, he gives new sense and meaning to the world. Following Du Toit (in the previous chapter), it is emphasized that [spoken] language develops into a system of symbols. Certain language sounds are associated with certain letter symbols. Later this understanding is extended to giving a particular number-name to a particular number-symbol. This linguistic understanding makes new meanings accessible to a child. To stress the importance of language, reference also is made to the previous chapters. Reading and interpreting the written word requires a greater distancing [to the cognitive] by the child because the speaker is no longer present or observable; only the characters are visible and he no longer hears the language but he has to give sound to the written language before he is able to "hear" the discourse. Arithmetic involves an additional degree of abstraction because the child not only has to be able to give the symbol a sound but also give the sound and written symbol a quantitative value or meaning. However, this matter is not discussed further here and the reader is referred to the current literature⁽²²⁾ on this issue.

Acquiring language and an arithmetic system are not isolated events, and both are profoundly connected with the child's entire becoming. Language is a medium for communicating, ordering, structuring, abstracting, thinking, actualizing intelligence, etc.; therefore, language derailment often is the origin of arithmetic difficulties.

The level of language acquisition greatly determines the level of arithmetic proficiency. According to Odendaal⁽²³⁾, "The effective

interpretation of language symbols is a necessary precondition for successful arithmetic achievement."

The arithmetic system is "carried" by the language system. This means it is an extension of the language system. To arrive at a good insight into the arithmetic system a child has to have an extensive vocabulary.⁽²⁴⁾ Whoever wants to evaluate a child's stagnation in learning arithmetic has to fathom his language proficiency. Often a language deficiency is the first symptom of an arithmetic difficulty.

6. Possible origins of arithmetic problems⁽²⁵⁾

Very seldom can learning difficulties be ascribed to one source. They can be brought about by intellectual, affective, normative and didactic factors in school or factors that originate in the pedagogic situation at home. Schonell⁽²⁶⁾ also says, "Backwardness in Arithmetic, as in other elementary school subjects, is usually due to a plurality of causes. The clear-cut case of backwardness which can be ascribed to a single cause is comparatively rare; the complex case which involves several causal factors is of common occurrence."

One or more of the following origins usually can be established with children who experience arithmetic difficulties. Although arithmetic problems usually are brought about by a complex of causes, for the purpose of this study they are divided into two main groups. The first group originates in pedagogic neglect while the second is a consequence of didactic neglect.

a) Deficiencies in preschool experiences

Children who have not had enough experience with concrete material usually experience a deficient acquisition of quantitative forms of language. These quantitative forms are acquired by dealing practically with concrete things. Inadequate experiences with practical things lead to a defective acquisition of language. Such pupils usually come from linguistically poor homes or surroundings. Thus, they experience a deficiency in the practical application of arithmetic concepts in everyday life. If a child hooks up with the abstract, the question arises if the concrete and the schematic levels [of thinking] possess adequate content on which he can fall back in searching for solutions to problems. A didactic error often made is that the visual is limited to beginning

arithmetic. Even in the higher grades it is necessary that new concepts such as fractions, area and proportions first be introduced visually. During these concrete experiences, the child's linguistic and intellectual potentialities are actualized.

Acquiring language and actualizing arithmetic potentialities are not isolated events but cohere with the child's total becoming [a person]. The connections between language and thought as well as between language and arithmetic are extremely important. Language and thought develop from a social visual level. This visual level contains the first experiences a child has acquired by means of his senses.

Van der Stoep⁽²⁷⁾ says that thinking develops more quickly than language but that language also is a channel for thinking. Van Gelder⁽²⁸⁾ says, "Thinking is guided by language, discovered through language and expressed in language." Thus, there is a close relationship and continual interaction between thinking and language. Without language, thinking will not develop and, conversely, without thinking, language stagnates. Van der Stoep⁽²⁹⁾ goes so far as to say that language and thought are inseparably related and that the level of language is actually the level of thought.

Thus, beginning arithmetic starts long before a child comes into contact with the formal system of arithmetic. First he handles things without the use of language but gradually he learns words of quantity and later numerals. Before he is able to work with numerals he has to know words that express the connection between quantities. Hence, language is a precondition for beginning arithmetic. The arithmetic system, then, can be viewed as a symbol system that originates from and is a form of language. Lamborn (Reason in Arithmetic) says, "Arithmetic, rightly regarded, is a language ... " and McSwain (A functional Programme in Arithmetic) says, " ... Arithmetic is a language for interpreting and communicating ideas of quantity and of relationships of quantities." Kwant (Phenomenology of language) views arithmetic and mathematics as extended forms of language.

Arithmetic rests on the language-values of the arithmetic symbols. A concept of quantity is meaningful only in light of the language meaning of that particular quantity. In his article (General language-

and reading-problems), Van der Stoep calls attention to the following language defects that can influence arithmetic achievement:

- i) **Inability to interpret symbols.** For many children the language symbol doesn't function as a symbol or a sign. For them, symbols do not represent concepts; and the letter-characters have no relationship to the living (spoken) language. With these children, we notice poor understanding and a low level of thinking. Consequently, for them word problems, in the first instance, are reading problems in the sense that the written symbols are not meaningful for them;
- ii) **Poor memory for letter-characters.** The names of the signs are frequently forgotten and some, such as 6 and 9, are often confused;
- iii) **Problems with analyzing the spoken word.** Children attach greater value to certain types of words such as, e.g., nouns and verbs. They neglect conjunctions and prepositions. In arithmetic, it is precisely these latter types of words that are very important because they indicate relationships. Since these words do not "speak" to him, the child doesn't see the relationships;
- iv) **Difficulty synthesizing.** The child cannot build up a word from its separate letter-characters;
- v) **Reading anxiety.** Anxiety arises in a child as a consequence of his **experiencing** his inability to achieve. Sometimes this is not a real inability to read but it is more an unsystematic mode of attack.

The following three types of error are prevalent in these cases:

- i) Confusion of the optical image of the word;
- ii) Hesitation and uncertainty about the form of the word and its relationship to sound, which then leads to reversals;
- iii) Incorrect meaning is given to the words.

Thus, it is clear that a child has to have a good vocabulary if he is to understand the arithmetic system. Consequently, good arithmetic teaching has to be paired with good language instruction. Gouws (Pedagogic evaluation of children with learning difficulties) says, "Whoever tries to explore a stagnation in learning--also in the case of arithmetic and mathematics--without investigating the availability of language really makes a futile effort because the primary symptom of disturbed learning is a disturbed use of language."

b) Learning formal arithmetic too early

As already indicated, a child first has to acquire adequate practical experience before formal arithmetic can begin. Formal work without furnishing this concrete level of thought leads to frustrations. Formal instruction begun too early usually leads to a mechanical acquisition of numbers and operations because the child has not yet had adequate experience with the concepts. Consequently, concept formation is not yet adequately promoted so the work cannot be carried out with insight. Then, most of the operations are mostly or entirely meaningless to the child; for him they are "school work" and not something realistic or a part of his life experiences and life world.

c) Poor school attendance

In discussing arithmetic as a subject, attention was called to the fact that it is a logical subject and that one fact is built on preceding ones. Deficient knowledge of the basic principles will "haunt" later work because the foundation on which it has to be built is then lacking.

Absence from school, whether a consequence of sickness, truancy, moving around by the parents or whatever reasons necessarily will lead to the child missing certain work. As a result of these absences, gaps arise in the structure of the child's knowledge. In arithmetic, these gaps are very important because not only does the child lose these parts of the work but he also is not able to understand the work that follows later and which is built on these specific, and missed facts. Thus, a situation arises in which the child falls always more behind because he cannot master the work as a result of the gaps in the content of his knowledge. He is unable to catch up and falls farther behind, and we have a situation of accumulated confusion. The result is discouragement, and such children also say that they no longer understand.

d) Discontinuity in the work

This cause is closely connected to the preceding one, and this discontinuity can occur in different stages of a child's becoming. As a result of discontinuity, gaps arise in a child's life experiences. These gaps can be found between the preschool and elementary

school work, usually because formal instruction is begun too soon (i.e., before the concrete level of thinking is adequately furnished).

These gaps can also arise between any other grades or even during the year as a result of poorly planned curricula work schemes. When intensive planning is lacking, certain work will be omitted. This missing knowledge is needed to solve subsequent problems. If, however, a child has not had the necessary experience and thus does not possess the required knowledge which can be transferred to the assigned task, the solution to the problem will fall outside of his field of experience and thus outside of his grasp; if he is expected to carry out the task and solve the problem, he will be forced to use a trial-and-error approach.

In switching from the primary to the secondary school, where there is a change from class to subject matter teaching, gaps also can easily arise in certain pupil's knowledge. In high school, pupils from different elementary schools are brought together in one class and it is assumed they have the same basic knowledge. From practical experience, knowledge in such a heterogeneous class is very diverse, and it is a healthy didactic principle to briefly repeat to all of the children the most important basic principles at the beginning of their high school career.

e) Errors in teaching arithmetic

Although many errors in teaching arithmetic can be indicated, the aim here is not to list them but to briefly indicate a few of the most important ones and, thus, no claim of completeness is made.

The first error is an exaggerated emphasis of the subject matter at the expense of the child. It is important that a child learn **arithmetic** but it is more important to remember that it is a **child** who has to learn it. Thus, the child should always be at the center of teaching and not the subject matter. A subject only has value in so far as it has value for a child. Hence, knowledge is only valuable if it is meaningful and useful to a person.

Also, the work schemes are sometimes planned according to the demands of the subject without considering the child. Sometimes the work tempo is entirely too fast for the weaker children with the result that they do not have an adequate opportunity to form

concepts, but they acquire for themselves artificial techniques which even allow them to achieve in the lower grades; however, usually in the higher grades their achievement is not maintained. There is too little consideration of the child's uniqueness and, thus, the emphasis should fall much more on each individual child with his particular problems.

An additional didactic problem stems from the teacher's instructions. The children are taught to use fixed patterns to complete the work. Sometimes more time is spent on firming up certain methods and explanations than on actual arithmetic. It often happens that correct solutions not found by particular methods are not accepted. The preferred didactic approach should be to guide the children to discover that most problems can be solved in different ways, and a very good practice is to allow them to experiment in order to determine how many ways a problem can be solved. If a pupil can solve a problem by his own methods, this is a clear indication that he understands the work. However, most pupils find solutions to problems with certain stereotyped methods. In many cases they have no understanding of the underlying basic principles. For them, problem solving is no more than applying particular techniques. Of course, the application of techniques is not summarily condemned; rather, the principles on which the techniques rest have to be made known to the children and understood by them. If a child understands the technique, he will fit it into his own pattern of thinking and use it to solve problems.

Still another didactic mistake arises from the excessive emphasis on the speed of carrying out the operations. The view that the person who most quickly executes the operations is necessarily the best at arithmetic has led to the overemphasis of mechanical drill without insight. Without any thought, a child can say that 9 plus 6 equals 15, but very seldom is able to explain why it is 15 and not 14 or 16 because he doesn't understand the principle on which the operation of addition rests. The overemphasis on speed and neatness is linked to the uniformity that is expected of all pupils. The consequence of this instruction is a deficient initiative and acquisition of methods by imitation. Thus, this really is schooling the child's memory instead of appealing to his original thinking.

f) Intellectual deficiencies

In many cases arithmetic problems are experienced because of deficient intellectual potential. Various researchers have indicated that the correct execution of arithmetic operations, especially with insight and understanding, requires a certain degree of intelligence. If a pupil does not possess this minimum potential, he is unable to solve arithmetic problems. Also, this deficiency cannot be improved. A less gifted person finds it extremely difficult to think abstractly. His low intellect "haunts" him more in solving problems than in routine work. In solving problems, a child is expected to generalize and apply known structures and systems to new situations. Often, a less gifted person is not able to make this application. Sometimes the less gifted will use hit-and-miss methods. He will use solution techniques and if he doesn't achieve success with one technique he will use another. Sometimes a solution is found by a fluke. However, for these pupils there is no insight into the problem, where for those with an insight into and understanding of the problem, the correct operations are immediately applied to find a solution.

The child's actualization of his intelligence also is extremely important. The question is whether he adequately actualizes his potentialities. Sometimes one gets the impression that a child has very low intelligence and yet, with further research, sometimes it is established that he has at his disposal a much higher intelligence but that he does not actualized his potentialities. This defective actualization can result from several sources, e.g., a deficient intention to learn which is spiritual in nature, or a weak physical state which leads to the child quickly tiring or other physical deficiencies such as poor vision, hearing or generally poor health.

The child's pedagogic situation also exerts a strong influence on the actualization of his intelligence because the pedagogic determines his directedness and his feelings of safety and security that, in turn, determine if he will be ready to venture into new situations. Therefore, it is very important that in any investigation of a child with arithmetic problems attention also has to be given to the pedagogic.

7. The learning (arithmetic) world of the child with arithmetic problems as an experiential world

Learning occurs by **experiencing**; the modes of learning are modes of **experiencing**; the learning child is an **experiencing** I and,

therefore, the learning world also is an experiential world. To say that the learning world is an experiential world means that the learning world (the arithmetic or quantitative world as formal arithmetic tasks) is a slice (momentary landscape) of reality as a greater whole with which a child, in a particular moment of **experiencing**, is intentionally involved, to which he gives sense and meaning, to which he directs himself pathically-effectively, gnostically-cognitively and normatively, with respect to which he takes a position, etc.

Although each child with arithmetic problems is viewed as an individual and his uniqueness is acknowledged, still a general image of these children is acquired from the current phenomenological literature as well as from the results of our own research. In this light, and in accordance with some current descriptions of the concepts **experiencing** and **experiential world**⁽³⁰⁾, the following are aspects of the learning (arithmetic) world, as experiential world, of a child with arithmetic problems:

a) Poor, confused or labile intention to learn

These children show a poor intentional directness toward the quantitative world which is unlocked for them in the formal situation of arithmetic teaching, in arithmetic problems given as homework as well as in the class test or examination situation. Entry into the unlocked quantitative world, as **experienced** content, is characterized by a weak venturing attitude along with an unwillingness (purposefully not willing) or superficial willingness. A readiness to learn is largely or entirely lacking. Through his **experiencing** the quantitative world the child has to transform it into a world-for-me; he has to acquire a grasp of the quantitative world; he has to master it and acquire a proficiency in arithmetic. However, the inadequate stake that these children have in doing this means inadequate or deficient **experiencing** that results in labile arithmetic achievement. The child's deficient unlocking himself to the quantitative world takes the form of a lapse in attending (e.g., daydreaming), poor concentration, deficient interests and an unwillingness to learn. A child is always openness and possibility but because of this poor, confused or labile intention to learn he isn't adequately open to what he has to acquire, and his potentialities are not actualized.

b) Attunement (taking a position, disposition)

It is precisely the attunement of these children that interferes with solving arithmetic problems and, therefore, a psychotherapeutic correction of this attunement is a precondition for overcoming his problems in learning arithmetic. His global, undifferentiated, strongly affectively colored attunement hinders his ability to direct his potentiality for analytic knowing to arithmetic tasks. This pathic-concrete disposition interferes with actualizing the act-structures underlying arithmetic because they require that he function on a higher cognitive, distanced level of structuring and abstracting. Knowing, ordering, analyzing and differentiating are inadequately actualized.

An unfavorable work attitude can hinder the child in overcoming the difficulties he encounters in acquiring the arithmetic system. A readiness (as affective readiness) to take on a task and work are lacking. In these children an attitude arises of avoidance and rejection of arithmetic tasks that are so problematic for them. Instead of a business-like independence regarding arithmetic tasks, he displays a self-concerned search for help.

The disposition of these children also points to an infantilism: they are too childlike (childish) for his age, and this influences both their pathic-affective and gnostic-cognitive ways of **experiencing**. Such a child refuses to grow up, to accept tasks and responsibilities. He prefers the form of existence of a younger child that is without obligations. He searches for what is pleasant and for what is immediately gratifying; he strives for freedom on a vital level. These modes of **experiencing** are paired with dependency and docility.

c) Giving sense and meaning

The quantitative world, as experiential world, also is a world of sense and meaning. As a world with many meanings, it is open regarding what feeling-, striving-, thinking- and value-meanings a child will attribute to it. The question, then, is what sense and meaning does a child with arithmetic problems give to the quantitative world and to his own inability and unwillingness to master it? The sense and meaning given to the arithmetic system are usually deficient, negative and unfavorable. His failures lead to his **experiencing** the arithmetic system as senseless and without

value; he **experiences** mastering it as a goal of little value and as unattainable.

The existential-ethical concern of these children with the quantitative world is not responsible. This raises the question of their normative **experience** of sense, i.e., their striving- and value-meanings with respect to arithmetic as well as themselves. The arithmetic system and the self are **experienced** as **inferior in value**, his learning conscience nags him and he **experiences** an existential unrest because his **striving** for life fulfillment is nipped in the bud by his problem with arithmetic. Such a child asks and seeks for the meaning of his own arithmetic problems and, therefore, of his own existence; he looks for the meaning of his own fate and destiny as a child with arithmetic difficulties, and for a sense of his own attempts and failures; he intensely hopes and expects to fulfill his own ability to be able to know arithmetic with proficiency.

d) Pathic-affective lived-experiencing

Here the concern is with the child's emotional meanings, i.e., with how he **feels** about his problem with arithmetic. His pathic-affective **experiencing** restrains, to a greater or lesser degree, his acquisition of arithmetic proficiency. A predominantly pathic disposition and labile affectivity characterizes such a child's directedness to the quantitative world which results in labile behaving and achieving in arithmetic. A disordered, confusing and negative pathic **experiencing** means for the child a pathic flooding which provides an unfavorable foundation for the cognitive **experiencing** desired. (Thus, problems with learning arithmetic are clearly problems of pathic-affective **experiencing**).

On the one hand, a child is restrained by his pathic **experiences** of his problem with arithmetic (it is difficult, unpleasant--he hates it); on the other hand, he **experiences** his own impotence which is paired with feelings of apathy, antipathy, inferiority, deficient self-confidence, restlessness, learning anxiety, over-sensitivity to criticism, insecurity, uncertainty, discontentment, discouragement, threat, embarrassment, desperation, distress, doubt, disappointment, despair, etc. Such children, then, are quickly upset and flooded pathically which again leads to a flooding and disorientation of their gnostic-cognitive **experiences**.

Also, it is usually the case that there is a vicious circle between a child's affective disturbance and his problem with arithmetic. Affective disturbances so flood him that his arithmetic problem is worsened. His **experience** of this and of his increased impotence plunge him further into the affective distress with which he is laboring.

In addition, such a child usually feel guilty (as primarily a pathic-affective **experience**) about his failures in arithmetic. He feels guilty because, through no fault of his own, he fails in school. Even if he is not blamed, he is still inclined to compare himself with a measuring stick external to himself (in terms of what others can do) or with an internal one, with himself. For example, he feels that he really can do well but doesn't live up to his own measure. Feelings of dissatisfaction, inferiority and guilt about his achievement are closely related to each other. The child **experiences** an existential restlessness because he has the idea that he cannot wholeheartedly fulfill his own destiny.⁽³¹⁾

e) Gnostic-cognitive lived-experiencing

Here the concern is with a child's cognitive meanings regarding his problem with arithmetic. In other words, what does he **think** of his arithmetic problem? Although such a child has an idea of his own ignorance and is aware of his own inability to acquire the arithmetic system, his **experience** of the learning problem is clearly an emotional one. Pathic **experiences** flood such a child so that he has difficulty distancing himself from his own problem and therefore assimilates and accepts the problem on a gnostic-normative level in deficient ways.

f) Modes of learning as modes of lived-experiencing

The pathic-affective flooding of these children means an inability to distance themselves to a cognitive level of **experiencing** which is required for them to actualize their modes of learning*, namely,

* In contemporary psychopedagogic thought **sensing** is the pathic-affective mode founding all other modes. **Attending** is a transitional mode to the gnostic-cognitive modes of **perceiving, thinking, imagining and fantasizing** and **remembering**. Actualizing intelligence is of a different order in that it involves all

sensing, perceiving, imagining and fantasizing, thinking, remembering and actualizing intelligence.⁽³²⁾ They show an image of disturbed concentration, disordered thinking, poor cognitive directedness, underactualized intellectual potentialities as well as a narrow sphere of **experiences**. Notwithstanding the arithmetic problem itself, these children are not able to sufficiently distance themselves to the quantitative world and take an objective-cognitive perspective; by renouncing the quantitative reality, they are not able to achieve in arithmetic.

These children do not acquire an insight into the arithmetic system. According to Van Hiele's⁽³³⁾ definition, they do not direct themselves adequately in new arithmetic situations. On the contrary, new arithmetic situations horrify these children and they recoil from them.

Because of their lack of a gnostic-cognitive disposition, these children find it difficult to order their **experiences** in formal systems (as cognitive means). Exploring and participating in the quantitative world are deficient. Regarding their acquisition of arithmetic, these children do not show that they want to be someone themselves.

More specifically, regarding the act-structures underlying arithmetic, these children are not able to arrive at a higher cognitive level in order to build up relationships, structures and abstractions. They are unable to deal with particular methods of solution and operational schemes. There are disturbances in simple operations as well as in following rules mechanically.

For them, stagnation, disintegration or slowness characterize learning arithmetic.

g) Learning relationships as lived-experienced relationships

A child acquires the arithmetic system in relationships with himself, other children, adults (parents and teachers) and things (quantitative world).

of the modes of learning to break through situations to new insights, understandings and possibilities. G.Y.

Regarding his relationship with himself, a child with arithmetic problems **experiences** dissatisfaction with himself with the possibility of not accepting himself. Also, he **experiences** conflict because, on the one hand, he is inclined to avoid obligations and, on the other hand, he longs to achieve in arithmetic.

These children also **experience** their shortcomings through the eyes of others. Everyone sees them as children with arithmetic problems. This leads to isolation (from others as well as from the quantitative world) and to aggression (often in the form of conflicts with authority and a relationship of revolt). The subject matter and the teacher of arithmetic are rejected. Because of his pathic directedness toward arithmetic tasks, his contact with the quantitative world is deficient.

Further, these children often revolt against a society that makes difficult demands. This is the society (parents and teachers as far as a child is concerned) that demands that he becomes proficient at arithmetic if he is to be viewed as a full-fledged member of it. In a culture that does not demand arithmetic proficiency, a child will not be considered as a child who is derailed in arithmetic.

Others also see that he cannot acquire proficiency in arithmetic, and they devalue him because of this. A child with arithmetic problems accepts this judgment that he is of less value (inferior) and, thus, he devalues himself. He **experiences** himself as inferior and **different** from others. This negative pathic **experience** includes an oversensitivity to the "hidden, masked, negative feelings of others", which he "**experiences** as depreciation."⁽³⁴⁾

h) Deficiencies in lived-experiencing

The image of the lived experiences of these children includes aspects of deficient **experiencing** (especially the gnostic-cognitive but also the pathic-affective and the normative), defective **experiences** of cognitive potentialities, deficient, negative or unfavorable ways of **experiencing** and an underactualization of **experienced** potentialities. This image refers to the child's restrained becoming--as a result of his problem with arithmetic, the course of his becoming is characterized as retarded, stagnated, obstructed, failing and difficult.

8. Orthopedagogic-orthodidactic evaluation of children with arithmetic problems

a) Acquiring a learning image as image of the lived-experiences of a child with arithmetic problems

First of all, the orthopedagogic-orthodidactic evaluation of children with arithmetic problems entails acquiring a learning image, as image of the **experiences** of these children. Such an image includes the following: an image of the child's habitual directedness to the arithmetic tasks; of his habitual opposition to them; of the sense and meaning he gives to and receives from the quantitative world; of his arithmetic problem and his inability; of his pathic-affective, gnostic-cognitive and normative meanings of the arithmetic tasks; of his modes of learning as modes of **experiencing**; of his learning relationships as **experienced** relationships; and of his defective **experiencing** and disturbed modes of **experiencing**.

The ways in which this learning image is acquired, including the approaches, procedures, descriptions of media used, etc. is not discussed in detail here. In this connection, the reader is referred to Chapter V by Kotze and more specifically to the work of Odendaal⁽³⁵⁾ regarding children with arithmetic problems.

However, briefly noted, the orthopedagogue-orthodidactician acquires a learning image by means of a pedagogic encounter, discourse and observation, by attaining an historicity image--this includes a history of the child's history of his past, present and future as he **experiences** them--and by using the following exploratory (learning) media: observation media such as the Koh's blocks, "Guide-it", form media; projective media such as the T.A.T., Columbus, S.A.P.A.T., Four-picture medium of Van Lennep; the incomplete sentences (after Rotter); language media (see below); graphic-expression (drawing) media such as the Wartegg, draw a person, a house, a tree; intelligence media (not only to attain an IQ but especially to perform a quantitative analysis of the child's intelligence as well as a pedagogic evaluation of the child's actualization of his intellectual potentialities.

b) Acquiring an orthodidactic image of a child with arithmetic problems

Because of the close connection between language difficulties arithmetic problems, an orthodidactic image has to include a language image* and an arithmetic image. Language media (composition, reading and comprehension tests, spelling, sentence completion) have to be used in order to fathom the language acquired to determine if the stagnation in acquiring the arithmetic system isn't already evident in the level of the child's acquired language.

An arithmetic image is attained by using **diagnostic arithmetic tests**.⁽³⁶⁾ An **arithmetic (word) problem test**⁽³⁷⁾ needs to be included. The acquisition of an arithmetic image does not merely involve applying scholastic and diagnostic arithmetic tests to quantitatively evaluate a child's arithmetic performance or analyze his errors. The primary concern is examining, analyzing and evaluating his arithmetic achievements to obtain an orthodidactic image, as an image of his difficulties with arithmetic. Analyses of these difficulties are done on the basis of the available knowledge about the arithmetic-learning event. Then the following **arithmetic act-structures** (or activity forms) have to be fathomed as they themselves appear in the constitution of the arithmetic tasks:

- i) the non-cognitive arithmetic act-structures (automatisms);
- ii) cognitive arithmetic act-structures:
 - concretizing** (e.g., on the level of beginning arithmetic);
 - schematizing** (e.g., the scheme for long-division);
 - abstracting** (e.g., solving arithmetic [word] problems by thinking and conceptualizing and by using abstract concepts and relationships⁽³⁸⁾).

The arithmetic image of the child with arithmetic problems, then, includes the following aspects:

- i) analysis of his intention to learn regarding arithmetic tasks;
- ii) qualitative error-analysis of the above arithmetic act-structures. The nature of the errors, faulty steps, operations, procedures, etc. is fathomed;

* In this connection, see du Toit's discussion of evaluating language problems in Chapter VI.

iii) an image of arithmetic difficulties in terms of deviations in constituting the arithmetic tasks.

This image of arithmetic derailments will then serve as the points for linking up with the orthodidactic assistance the child has to receive regarding his arithmetic problems.

9. Orthopedagogic-orthodidactic assistance for children with arithmetic problems

a) Orthopedagogic help or pedotherapy

Once again it is stressed that assisting a child with arithmetic problems in an educationally accountable way is primarily an orthopedagogic matter while orthodidactic assistance, as corrective teaching, is secondary to this. It is a child as a totality-in-communication who has failed the arithmetic tasks and not his brain. Also, as a totality-in-communication, he **experiences** his failure. Therefore, assistance has to be directed to his total situation and this involves, first, supporting him to enter and assimilate his disturbed learning (arithmetic) world, as an experiential world. He has to be supported pathically-affectively, gnostically-cognitively and normatively in his interpretation of his difficulties so that he can adequately direct himself to the arithmetic tasks with the desired attitude toward arithmetic and his own potentialities. Deficient **experiences** have to be further replenished and elevated, and he has to be supported in actualizing his cognitive modes of **experiencing**; learning relationships have to be corrected, etc.

Therefore, orthopedagogic assistance has to occur according to the learning (arithmetic) image obtained from the child; in other words, this is accomplished by analyzing and penetrating the phenomenon child-with-arithmetic difficulties in terms of the category lived experience. Now that the question is answered about how he **experiences** and explores the quantitative world and the different problematic image of a child with arithmetic problems is acquired, he has to be supported with the aim that he actualize his potentialities.

Further, orthopedagogic assistance has to be viewed as supporting a child to re-**experience** and re-define (pathically-affectively, gnostically-cognitively and normatively) his own situation--this is re-educating, re-orienting, re-directing and re-positioning him to

accept himself and his situation. The orthopedagogue-orthodidactician has to intervene in his **experiencing** the quantitative world and transform it into a more adaptable and attainable world for him. To him, the quantitative world is strange, threatening and abstract and he has to **experience** it as affectively safe and secure before he will want to venture in it.

Also, regarding the arithmetic tasks, the child has to be led to the existential moment of actualizing possibilities and in this way overcome his existential restlessness.

b) Pedagogically accountable orthodidactic assistance

These means of assisting are compiled according to the orthodidactic (arithmetic) image acquired by evaluating the individual child with arithmetic problems, and these means are linked up with the difficulties in the activity forms (act-structures) underlying arithmetic. That is, this involves correcting the faulty arithmetic activity forms and supporting the child to cognitively **re-experience** a particular slice of the quantitative world. Thus, this world has to be unlocked again for him so he can acquire a firm grasp (understanding) of it.

The orthopedagogue-orthodidactician has the task of compiling corrective orthodidactic methods as detailed elaborations of current didactic methods and which are linked directly to the child's arithmetic difficulties.

To encourage the child, to give him self-confidence and to help him create a distance from the arithmetic task, assistance has to begin on a level lower than what he is capable of. Clear rules, assignments, methods, structures, systems and aids for ordering have to serve as firm, trustworthy (and, therefore, as affectively warm) bases for him to acquire the arithmetic system.

10. An example of a learning image as image of the lived-experiences of a child with arithmetic problems

Sarie, 11 years, 9 months, grade 6

a) Historicity

Sarie was brought to the Child Guidance Clinic, Faculty of Education, University of Pretoria, by her mother because she experiences serious problems with arithmetic. She is averse to all schoolwork, cries easily when she is compelled to do it and then refuses to complete it. She does not willingly accept and complete **any** task. Yet she shows good progress in reading and spelling.

Sometimes she is impudent and also very childish. According to her mother, she **experiences** herself as unacceptable and rejected. She feels that her parents don't love her and sometimes accuses them by saying, "You don't love me."

i) Family situation

Sarie is the oldest of four children. Although family relationships are basically good, her mother is an extremely helpless, affectively disturbed, indulgent person, and Sarie does not accept her authority. Her father is much too strict and is inclined to make demands that are too high. Sarie demands more attention than the other children, especially from her mother. Her parents acknowledge that she possibly is able to **experience** their rejection of and disappointment in her. She has always received most of the attention from them.

ii) Developmental history and physical condition

At birth, Sarie did not receive oxygen immediately and, thus, there is the possibility of slight brain damage. According to her parent's, a psychiatrist's investigation was negative. He reported that she obtained an IQ of 44.

Sarie's further development was normal. However, for her age, she is very dependent, helpless, irresponsible and reliant. She usually relies on her younger sister. She is an uncertain and insecure who inadequately explores with her peers or other adults. Usually she refuses a request or demand point blank and then others usually give in to her.

Sarie's acquisition of language was normal, and she is very fluent bilingually. She is a reluctant and obstinate child. She has no friends, and she is untidy and disorderly as far as appearance and behavior are concerned.

iii) School situation

Sarie was 5 1/2 years old when she started school, and she was not school ready. She failed grade one because of her poor achievement in arithmetic, and she has never passed arithmetic. Her reading and spelling were always good; in her subjects of study (e.g., social studies), her achievement is satisfactory. She does not get along with her teachers and in school she also seeks acceptance and attention.

b) Pedagogic observation

Here Sarie's extreme obstinacy and hesitation to accept and complete a task (problem) stands out; the result is that she has an inadequate stake in any tasks that present her with gnostic-cognitive demands. Strong **experiences** of frustration and impotence are noticeable. She appears to be quiet, reticent and reserved; clearly, she is severely flooded pathically and she seems very confused, insecure and uncertain. Also, she shows a very weak venturing attitude as well as an extremely poor intentional directedness.

c) Implementing exploratory media*

"Guide-it": she has poor insight into this medium; she stagnates at the first obstacle. This is attributed to her poor directedness. In addition, she is right-handed and her movements do not appear to be conspicuously haphazard.

Form media: perceptual, motor and spatial disorders are conspicuous in her performance on the Ellis Visual Design, the Vedder-Figures and S medium.

Laterality media: according to these media Sarie is completely right-sided, and there is no disturbance in dominance or confusion in laterality. Consequently, her left-right orientation is in order.

Intelligence medium:

Quantitative analysis: on the N.S.A.I.S. Sarie obtained an IQ of 58 (V = 71, NV = 51).

Qualitative analysis: her performance is extremely labile (sub-scale points varied from 1 to 9). Her performance on the non-verbal items is remarkably weak. For example, on Pattern Completion, she had no items correct, and on Block Design, she had

* For documentation regarding the media, see Chapter V.

only the first item correct. Qualitatively, her intelligence is very weak. She stagnates at the slightest cognitive requirement and she is confined to an extremely concrete level.

Pedagogic evaluation: Sarie is not being educatively guided to responsibly actualize her intellectual potentialities. Her unsympathetic father's ambivalence as well as her over-sympathetic mother provide an extremely unfavorable foundation for adequately actualizing her intelligence. In terms of the language media (see below), it is clear that Sarie's **intelligence** [as potential] is **much higher** but that she severely underactualizes it [as achievement].

Projective media:

Rorschach Inkblot medium: Sarie's performance amounts to a refusal and again is a reflection of her extremely negative attitude. She was poorly directed to the task and gave only 11 responses of a global-diffuse nature: included were six cloud, three tree and one color response. Four were F (form) responses. Her performance indicates a pathic, labile attunement or disposition, serious affective disturbances, intense anxiety, flooding of gnostic-cognitive **experiences** by the pathic, uncertainty, insecurity, lability. Thus, her performance confirms her poor venturing attitude, poor directedness and averseness.

Columbus medium: her responses were merely concrete descriptions and she created no stories. Once again, she was remarkably poorly directed. There is one possible projection of a child who is punished because she is naughty.

Sentence completion: Sarie shows an above-average language potential and is reasonably well directed. According to this medium she **experiences** anxiety and tension, and she strives to succeed in school; she **experiences** her inability and failure in arithmetic; she seeks support and is dependent on her mother.

Graphic expression (drawing a person, a house, a tree):

According to the level, position, use of lines and background of the drawings as well as particular striking characteristics of them (e.g., erasures, proportions), an image was obtained of disorientation and disturbance. Here she shows an extreme infantilism; feels futility in striving to achieve; **experiences** conflict and intense anxiety, tension, lability, regression and a particular need for love and protection. There are further indications of a very basic uncertainty about life, a search for security and acceptance, a narrow life world and emotional poverty. Sarie's emphasis of the parts of the head in

the human drawing as well as particular erasures and deletions in the tree drawing possibly indicate brain injury.

d) Neurological examination

From the historicity image, particular form media, the block medium (item 7 of the N.S.A.I.S.) and the particular nature of her drawings, there is the possibility of slight brain damage.

Especially striking is her perceptual, motor and spatial disorientation. For these reasons, Sarie was referred for a neurological examination. The neurologist's findings briefly amount to the following:

Present problem

Her progress in school is poor and according to an investigation of her intelligence, she is retarded. Her arithmetic is extremely weak. she can only add and cannot multiply, divide or subtract. However, she can handle money. Numerical concepts are weak and she uses her fingers for counting. Her drawings are poor. Spelling and reading are apparently weak. She is entirely lost if she has to "read" a map. She cannot yet tie a knot, and she progresses poorly in sewing.

She is possessive of her mother, jealous of other children and wants to have constant attention and love. When a lot of attention is given to her, she becomes baby-ish in her speech and behavior. She cries easily, is moody and doesn't mingle easily with other children; especially in games, she starts arguments and complains. She is restless; her attention is easily distracted.

Her mother is affectively disturbed and has had a nervous breakdown.

The examination

Sarie was tearful and abnormally anxious. She was abnormally lively and hyperkinetic, sometimes with almost chorea-like (spasmodic) movements. Rhythmical movements were slow and uncertain. She walks awkwardly. She finds it difficult to imitate arm and leg positions. Her symptoms are poor, eye convergence is weak as is her grip.

E.E.G.: The record shows less than normal alpha activity and is more or less irregular with fluctuating rhythms and especially with irregular theta waves. No focus was indicated.

Conclusions

Sarie shows abnormal motor coordination, over-liveliness and, according to her historicity, poor attention and concentration. There is no doubt that she has neurological abnormalities that indicate cerebral abnormality, probably of a diffuse nature. Possibly the origin can be attributed to a lack of oxygen at her birth.

e) Orthodidactic image

i) Language image

Sarie reads very well and performs at a level above her age. She delivered an errorless reading performance; she reads easily, with reasonable intonations and phrasings. Insight into the content read is very good. Thus, there are no indications of reading defects or the defective interpretation of symbols.

She spells very well and shows no problems or weaknesses here. From a long list of 45 words, she misspelled only two. Also, on a concrete-everyday level, her composition indicates reasonable proficiency in structuring language. Here too there is no indication of impotence or deficiency with language.

Thus, Sarie's acquisition of language is on a level above that expected for her age; her language performance indicates a considerably higher intellectual potentiality than do the images obtained with the other media. Also, it is clear that Sarie's arithmetic deficiency is not a result of any language deficiency.

ii) Arithmetic image

Here Sarie shows an extremely weak intention to learn, a serious unwillingness and inability. She has mastered the basic arithmetic concepts (more/less, large/small, etc.); she knows her times tables quite well. Addition and subtraction combinations to 20 are very weak, especially subtraction. She is unable to deal with numbers, and she uses little tallies to add up and to subtract quantities.

Quantitative error analysis

Addition diagnosis: some of the errors of vertical addition are clearly due to her deficient mastery of the addition combinations.

Subtraction diagnosis: here there are numerous errors of oversight or lapses, for example:

$$\begin{array}{r}
 482 \\
 -142 \\
 \hline
 320 \quad (8 - 4 = 2)
 \end{array}$$

She is unable to carry the units to make subtraction possible; for example:

$$\begin{array}{r}
 706 \\
 -297 \\
 \hline
 588
 \end{array}$$

Multiplication, division and fraction diagnosis: here she shows a total confusion and inability, for example:

$$\begin{array}{r}
 603 \\
 \times 129 \\
 \hline
 5007
 \end{array}$$

Thus, she has no idea of how to handle the problem. With long-division and fractions, she doesn't get beyond the sums at the beginning.

Tests of arithmetic (word) problems

Here she is also totally confused. From the language text, she cannot concretize the arithmetic operations. Where a package of 96 pieces of candy is to be divided equally among 6 children, she presents the following:

$$\begin{array}{r}
 96 \\
 +6 \\
 \hline
 102
 \end{array}$$

Thus, it is clear she added instead of divided. She is not able to infer from the arithmetic problem which operation she should apply. She cannot correctly carry out the operation that she does choose. Here she fails completely.

Image of arithmetic difficulty

An extremely poor learning intention and intense pathic flooding characterize Sarie's constituting the arithmetic tasks. These aspects are also central to her total inability in arithmetic. She rejects arithmetic tasks outright. The arithmetic material directs an extremely negative appeal to her. The arithmetic intention, which is a precondition for cognitive arithmetic act-structures such as active structuring (planning), system forming, seeing relationships, etc., are almost entirely absent. She knows her times tables (automatizations), but is not thereby able to see into the connection between, e.g., the multiplication and the division tables. True insight into the structure of arithmetic is lacking.

From Sarie's arithmetic performance, it is clear that she is confined to a concrete level of thinking. There is no mention of schematizing, abstracting, problem solving, actualizing arithmetic products and maneuverability (transfer) of arithmetic act-structures. She has not yet mastered the systems of long-division, long-multiplication and fractions.

f) Summary of the learning image, as image of lived-experiences, for Sarie

Task unwillingness, serious retardation in arithmetic, a strongly pathic orientation and a general disorientation are prominent aspects of Sarie's learning image. In particular, regarding arithmetic tasks, she usually shows a confused, labile, pathically flooded intention to learn. Not only is her poor arithmetic achievement evident but so is a serious **underactualization of her intellectual potentialities**. Her attention is easily distracted and she shows an inability to concentrate. Her weak venturing attitude is paired with a conspicuous confusion in thinking and in her emotional life. With Sarie, there is little understanding of the quantitative world.

Her attitude toward arithmetic is so negative and is on such a strongly pathic-concrete level that she is completely stagnated in acquiring the arithmetic system. Namely, she manifests an habitual global-diffuse mode of **experiencing** (i.e., lived-experiencing). Also, she is so self-rejecting and so oriented to seeking support that she is unable to handle arithmetic tasks and to overcome any obstacles.

An image of extreme pathic and gnostic infantility was obtained: she is very dependent, not-responsible, helpless and reliant for her age; she cries easily and is childish in her behavior and ways of **experiencing**. Her emotional, thinking, valuing and striving interpretation of herself and of arithmetic tasks are extremely unfavorable, negative, deficient and disturbed.

Sarie's serious affective disturbance includes a strong pathic disposition and the **experience** of impotence and failure; her labile affectivity points to pathic flooding, to a pathic need for love, protection and acceptance, to antipathy, to confused and bewildered **experiences**, to intense anxiety (also anxiety regarding learning), to intense feelings of insecurity and a basic uncertainty about life. Also she feels threatened by the too high demands placed on her. There is a clear indication of a vicious circle between her affective disturbance and her problem with arithmetic such that there is an image of extreme affective distress.

She is aware of her inability to acquire arithmetic proficiency, but she is not able to distance herself from her problem; even less can she accept and assimilate her problematic situation. Also, her striving to achieve in arithmetic is futile.

As a consequence of her poor learning intention and pathic flooding, the modes of learning and, specifically, the cognitive act-structures are not actualized adequately. Thus, she does not actualize her intellectual potentialities; also, her intelligence is qualitatively very weak while a pedagogic evaluation of her actualizing her intelligence indicates that she is not being supported (educated) affectively, cognitively and normatively to responsibly actualize her intellectuality. Poor achievement, extremely deficient insight and a serious stagnation characterize her acquisition of proficiency in arithmetic.

Sarie **experiences** herself as a failure, as unloved and unaccepted. She is quiet, reserved and private, and she is not popular with other children or adults. She is jealous of other children and excessively seeks love and attention.

Ambivalence characterizes her learning relationship to her parents: regarding her indulgent mother, she is possessive of but also aggressive and obstinate toward her--she does not accept her

mother's authority; regarding her over-strict father with demands that are too high for her, she **experiences** anxiety, uncertainty, discouragement and confusion. Also, her relationships with her teachers are poor.

Sarie has an attitude of rejection of and aversion for the quantitative world (and more specifically, arithmetic as a subject). Her contact with the quantitative world is a deficient one. Her attenuated dialogue with this world points to deficient pathic-affective, gnostic-cognitive and normative **experiences** and to disturbed ways of **experiencing** and, consequently, to a restrained becoming, difficult being-on-the-way to adulthood.

Her dyscalculia is mainly the result of pedagogic deficiencies (serious pedagogic neglect), affective disturbances, psychic disability and a slight neurological deviation.

Her language proficiency is on a level above that expected for her age, which indicates an above-average intellectual potentiality and which also shows that her problems with arithmetic are not attributable to language deficiencies. In light of the favorable image of language, her IQ of 58 indicates how seriously her intelligence is **underactualized**. Although her apparently low (functioning) intelligence and the slight brain damage might be possible factors in her difficulties with arithmetic, an evaluation of Sarie's language indicates that they are not primary factors, and it is concluded that the primary factor is her extremely unfavorable pedagogic situation.

The image of Sarie's derailments in arithmetic includes indications of her weak intention regarding arithmetic, her strong pathic interpretation of arithmetic tasks and her rejection of arithmetic as a subject. She is totally impotent, especially with division, multiplication, fractions and arithmetic word problems. She stagnates on an extremely concrete level of **experiencing**; her grasp of the quantitative world is extremely deficient and her acquisition of the arithmetic system is generally stagnated. Although she has mastered addition automatisms, she is not able to actualize the arithmetic act-structures. She is able to **concretize** (e.g., simple addition and subtraction); however, she cannot **schematize** (e.g., long-division, long-multiplication, fractions) or **abstract** (e.g., arithmetic word problems).

g) The orthopedagogic-orthodidactic task regarding Sarie

The orthopedagogue-orthodidactician has to direct himself to Sarie's total situation and to her extremely disturbed **experience** of it. His primary task is to change this situation into a more assimilable one for her. First, he has to adjust her learning relationships, as pedagogic relationships, by discussions with her parents (and teachers). Sarie has to **experience** the necessary certainty, security, love, warmth, sincere interest and attention from her parents; she needs to receive extremely sympathetic but decisive authoritative guidance from them. With respect to her unwillingness, demands and values have to be presented to her; with respect to her stubbornness, prohibitions have to be laid down.

Sarie's intense **experience** of insecurity, her disorientation and inappropriate attunement (pathically flooded) include difficult orthopedagogic tasks in the form of pedotherapy, more specifically in the form of image therapy. Thus, helping Sarie implies **re-educating** because she has become derailed in her being-on-the-way to adulthood, **re-experiencing** as **re-defining** because her interpretation of her own situation and of arithmetic tasks is so disturbed and unfavorable, **re-attuning** because her focus is so extremely negative and **re-orienting** her learning intention because it is so extremely deficient.

Orthodidactic assistance has to be directed to changing the quantitative world into a more assimilable and realizable world for Sarie. Therefore, the orthopedagogue-orthdidactician has to create a pleasant, calm, tranquil, safe atmosphere within which the assistance occurs. He needs to encourage Sarie and accept her. He has to be sympathetic yet very decisive. To give her encouragement and self-confidence, help has to begin on a level on which she can achieve, namely, that of beginning arithmetic. Initially, extremely concrete procedures need to be followed. First, the addition and subtraction combinations to 20 have to be practiced thoroughly and linked up with concrete examples of the relationship between number symbols and quantities and between different relative numbers. Second, Sarie needs to be systematically helped step-by-step to acquire the systems of addition, subtraction, multiplication, division and fractions. Finally, by very simple examples, arithmetic word problems are introduced. As her self-confidence and insight increases, more difficult, abstract tasks can be presented to her until she attains the desired level of proficiency in arithmetic and it is

clear that she optimally actualizes her intellectual potentialities in general and also with respect to arithmetic tasks.

11. Examples of arithmetic difficulties and the orthopedagogic-orthodidactic assistance of children with such difficulties

With the aim of providing help for pupils with problems in arithmetic, difficulties will be shown with examples taken from the work of pupils. Assistance, then, consists in replenishing the conspicuous deficiencies.

Pupil A

This pupil is in grade 2. She is 7 years 8 months old and is the oldest in a family of four children. Home circumstances are healthy. In grade one, she achieved well in arithmetic, but during the first semester of grade two, the problem arose. Not only her arithmetic but also her handwriting deteriorated. From the investigation, it was clear that she is entirely uncertain and, consequently, is too afraid to venture.

Didactically, it appears that she does not understand the decimal borrowing system, that is, when she has to transfer ten to the unit column from the tens column. She had tried all possibilities but her work was always incorrect; sometimes she is ridiculed because she is not able to do the "easy sums"; she **experienced** her failures with gradually increasing intensity with the result that she already shows signs of refusing to communicate with her surroundings.

For this child, orthopedagogic assistance is given first. By means of play, contact is made with her and her trust is won. Orthodidactic assistance followed this. With concrete objects, the concept of grouping is clarified for her. Later, these concrete groupings are related to the number system. This once again shows that orthopedagogic-orthodidactic assistance are not viewed as separate approaches but rather as of supplementary importance to the child as a total being.

Problems with carrying are also found among different pupils. **Pupil B** (grade four) did not carry with multiplication sums as the following examples from her work clearly show:

$$\begin{array}{r} \text{x4} \\ \hline 160 \end{array}$$

$$\begin{array}{r} \text{x9} \\ \hline 811 \end{array}$$

For this pupil orthodidactic help also lies in the concept of grouping. Transferring a group of ten from the tens (column) has to be handled practically.

Pupil C also experiences problems with carrying as in the following operations:

$$\begin{aligned} 93 + 88 &= (90 + 3) + (80 + 8) \\ &= (90 + 80) + (3 + 8) \\ &= 170 + 11 \\ &= 170 + \boxed{10} + 1 \quad \text{not included} \\ &= 171 \end{aligned}$$

With these children, the structure of addition has not been adequately acquired.

When operations are carried out with common fractions, in many cases the errors are a direct consequence of a deficient knowledge of the structure of fractions. From the work of a number of pupils it is clear that most of their difficulties were experienced in converting the fractions, e.g., $1/2$ converted into eighths.

Few or any of these pupils were aware of the role of the identical (common) element. In all cases, conversions or transformations had been learned mechanically. Thus, for them it was only a technique. The denominator of one fraction is only divided into the denominator of the new fraction and the quotient is then multiplied by the numerator. They have no idea why this is done with the consequence that the operation has become entirely mechanical. The following errors appeared in the work of a pair of fifth grade pupils:

$$\begin{aligned} 1/8 + 3/4 &= \\ &\rightarrow = 1/8 + 3/8 \\ &= 4/8 \end{aligned}$$

$$1/2 - 3/8 =$$

$$\begin{aligned} & \Rightarrow 1/8 - 3/8 \\ & = 2/8 \end{aligned}$$

$$\begin{aligned} 2/5 + 3/10 = & \\ \Rightarrow 2/10 + 3/10 & \\ = 5/10 & \end{aligned}$$

For these pupils, orthodidactic help is in correcting the basic fundamental concepts. In adding mixed numbers, it is found that either only the plain numbers or only parts of the fraction are counted together. All of this indicates serious didactic neglect because the pupils continually make the same mistakes and they are not shown the correct procedure.

The most common error in subtracting fractions is to add instead of subtract. Even though minus signs appear in the problems, they commonly add. the following pupil is in grade 6:

$$\begin{aligned} \text{a) } 5 \frac{7}{8} - 2 \frac{3}{4} = & \\ = 7 \frac{(7-6)}{8} & \\ = 7 \frac{13}{8} & \\ = 8 \frac{5}{8} & \end{aligned}$$

$$\begin{aligned} \text{b) } 6 \frac{9}{10} - 4 \frac{3}{5} = & \\ = 10 \frac{(9-6)}{10} & \\ = 10 \frac{15}{10} & \\ = 11 \frac{5}{10} & \\ = 11 \frac{1}{2} & \end{aligned}$$

Either this pupil does not know the meaning of the minus sign or he doesn't pay attention to it because the same mistake was made repeatedly.

Orthodidactic assistance for these pupils includes repeating the meaning of the minus sign and stressing the difference between the operations of addition and subtraction.

When fractions are multiplied, a number of general errors is also observed. the following is from the work of a pupil in grade seven. He makes the same error throughout. He confuses multiplication with addition:

$$\begin{aligned}
 \text{a) } & 2/3 \times 15/19 \times 5/12 = \\
 & = (14 + 63 + 17)/48 \\
 & = 94/48 \quad \text{Lowest common multiple and} \\
 & \quad \text{reduction are also in error.}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & 3/4 \times 2/5 \times 1/8 = \\
 & = (31 + 12 + 17)/40 \\
 & = 60/40 \quad \text{Reduction also in error.}
 \end{aligned}$$

When mixed numbers are multiplied, the plain numbers and the fractions are multiplied separately. Here we have to attend to the faulty operation scheme used by this child:

$$\begin{aligned}
 \text{a) } & 2 \frac{1}{2} \times 5 \frac{3}{8} = \\
 & \quad \quad \quad \swarrow \quad \searrow \\
 & \quad \quad \quad \blacktriangleright \quad \blacktriangleright \\
 & \quad \quad \quad = 10 \frac{4}{8} \times \frac{3}{8} \\
 & \quad \quad \quad = 10 \frac{12}{8} \\
 & \quad \quad \quad = 11 \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & 2 \frac{1}{2} \times 1 \frac{3}{4} = \\
 & \quad \quad \quad \swarrow \quad \searrow \\
 & \quad \quad \quad \blacktriangleright \quad \blacktriangleright \\
 & \quad \quad \quad = 2 \frac{1}{2} \times \frac{3}{4} \\
 & \quad \quad \quad = 2 \frac{3}{8}
 \end{aligned}$$

All arithmetic is the correct execution of operations. Orthodidactic help thus will mean acquiring the correct schemes of operation.

The greatest confusion is experienced when the division operation is carried out with fractions. This confusion stems from the technique of "invert the divisor and multiply", without any basis for this being given or without the pupils having the least idea of why one has to do this. The following examples also all indicate the confusion that surely exists:

$$\begin{aligned} \text{a) } 6/(3/4) &= \\ &= 6/1 \times 3/4 \\ &= 3/24 \end{aligned}$$

The error was made throughout.

$$\begin{aligned} \text{b) } 8/(2/3) &= \\ &= 8/1 \times 2/3 \\ &= 2/24 \end{aligned}$$

The arithmetic operations are correct but the operation structure is faulty. Orthodidactic assistance thus lies in correcting the operation structure.

We found an additional confusion with another pupil. Here the divisor and the dividend are consistently interchanged:

$$\begin{aligned} \text{a) } 10/(4/5) &= \quad \quad 2 \\ &= 4/\overset{2}{\cancel{5}} \times \underset{1}{\cancel{10}}/1 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{b) } (112 \ 1/2)/(7 \ 1/2) &= \\ &= 15/2 \times 225/2 \\ &= \end{aligned}$$

These examples illustrate the existing confusion and the consequence is of poor didactic presentations.

12. Synthesis

Orthopedagogic-orthodidactic evaluation and assistance are not concerned merely with diagnosing and correcting **problems with arithmetic**. The **child** with arithmetic problems, as a pedagogically situated, **experiencing-I** is in the foreground here. Thus, the arithmetic problem centers around the pedagogic situation of the child and his learning (arithmetic) world as experiential world. Help is not merely directed to the arithmetic but also to the child's **experiences**. The problem with arithmetic is

also a problem of **experiencing** and, therefore, this assistance is clearly an orthopedagogic-orthodidactic matter.

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